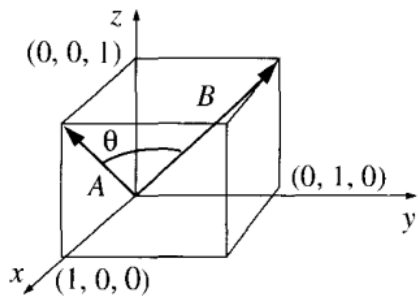
HW 1: **Vector Analysis**

Example 1.2 Find the angle between the face diagonals of a cube.



Problem 1.4 Use the cross product to find the components of the unit vector $\vec{n}$ perpendicular to the plane shown in Fig. 1.11.

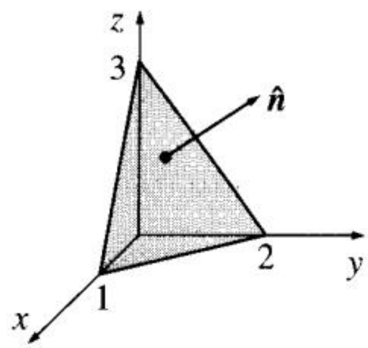


Figure 1.11

Problem 1.5 Prove the **BAC-CAB** rule by writing out both sides in component form.

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}).$$

Example 1.3 Find the gradient of $r = \sqrt{x^2 + y^2 + z^2}$ (the magnitude of the position vector).

Problem 1.15 Calculate the divergence of the following vector functions:

(a) $\mathbf{v}_a = x^2\,\hat{\mathbf{x}} + 3xz^2\,\hat{\mathbf{y}} - 2xz\,\hat{\mathbf{z}}$.

(b) $\mathbf{v}_b = xy\,\hat{\mathbf{x}} + 2yz\,\hat{\mathbf{y}} + 3zx\,\hat{\mathbf{z}}$.

(c) $\mathbf{v}_c = y^2\,\hat{\mathbf{x}} + (2xy + z^2)\,\hat{\mathbf{y}} + 2yz\,\hat{\mathbf{z}}$.

Problem 1.18 Calculate the curls of the vector functions in Prob. 1.15.

Problem 1.39 Compute the divergence of the function

$$\mathbf{v} = (r\cos\theta)\,\hat{\mathbf{r}} + (r\sin\theta)\,\hat{\boldsymbol{\theta}} + (r\sin\theta\cos\phi)\,\hat{\boldsymbol{\phi}}.$$

Check the divergence theorem for this function, using as your volume the inverted hemispherical bowl of radius R, resting on the xy plane and centered at the origin (Fig. 1.40).

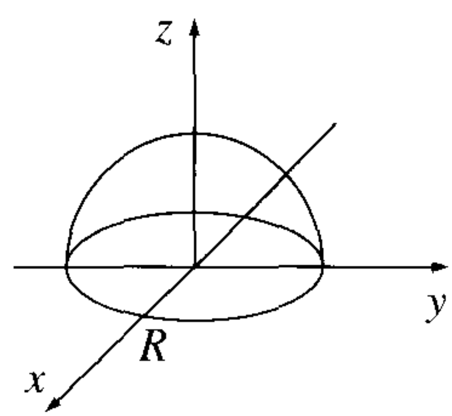


Figure 1.40